Chapter 11 Thermohaline Processes in the Ocean

Heating and freshening at the ocean surface produce a buoyancy flux that can result in density gradients in the upper ocean. Depending on the scale of the temperature and salinity gradients in the ocean, *thermohaline* transfer may occur by molecular diffusion (such as in the millimeter layer right below the air/sea interface), turbulent mixing (in the case of the ocean mixed layer), internal mixing processes, or by large-scale advective transport. Under certain circumstances, the ocean surface buoyancy flux can give rise to deep oceanic convection. The circulation that is driven principally by the surface buoyancy flux is called the thermohaline circulation.

11.1 Radiative Transfer in the Ocean

In Section 9.1.1, we showed that the surface albedo of the ocean, α_0, is typically about 5%, with values increasing for higher solar zenith angles. Therefore, greater than 90% of the solar radiation incident on the ocean is absorbed. At wavelengths greater than 0.8 μm, essentially all radiation is absorbed in the top 1 m and a substantial fraction is absorbed in the top 1 cm. By contrast, visible radiation can penetrate to depths in the ocean exceeding 50 m (Figure 11.1).

Extinction of solar radiation in the ocean can be approximated by a flux form of Beer's law (10.9):

$$F_\lambda(z) = F_\lambda(0)\left(1 - \alpha_{0\lambda}\right)\exp\left[-k_\lambda^{v,ext}\, z\right] \tag{11.1}$$

where $F_\lambda(0)$ is the shortwave radiation incident at the surface, z is depth below the surface, and $k_\lambda^{v,ext}$ is the volume shortwave extinction coefficient of the ocean which varies with wavelength (λ). The value of the absorption coefficient is increased relative to clear water if the water is turbid. In *turbid* water, selective absorption occurs by chlorophyll and other biotic pigments, suspended sediments, and dissolved matter. Away from coasts, it is principally the chlorophyll content that affects water clarity. Absorption at the near infrared wavelengths is insensitive to turbidity, while absorption in the visible wavelengths depends crucially on the chlorophyll content.

Solar radiation is the dominant heat source of the ocean. However, the rate of solar heating of the ocean is slow [see (3.33)], because of the large heat capacity of the ocean. In the tropics, where the net surface radiation flux may be as large as 1000 W m^{-2}, the heating rate of the upper 10 m of the ocean is only about 0.036°C hr^{-1}. Although the heating rate is not large, the cumulative effects of solar heating on the ocean is very important. The amount of solar energy absorbed by the oceans is about three times as large as that absorbed by the atmosphere. The heating of the ocean mixed layer by solar radiation is later transmitted to the atmosphere by the surface turbulent heat fluxes and longwave radiation. The portion of the solar radiation that penetrates below the ocean mixed layer can be transported over large horizontal distances by ocean currents.

11.2 Skin Temperature and the Diffusive Surface Layer

The temperature at the interface between the atmosphere and ocean is called the *skin sea surface temperature*. It is this interfacial temperature that is used in Section 9.1 to calculate the surface sensible and latent heat fluxes and the upwelling longwave flux. It is difficult to measure the skin temperature directly, and remote infrared thermometers must be employed. So-called sea surface temperatures are most commonly measured from ships with thermometers by sampling water at a depth of about 5 m from engine water intake, or from buoys or moorings that measure temperature at a depth of about 0.5 m. These measurements are referred to as *bulk sea surface temperatures* and are typically characteristic of the temperature of the ocean mixed layer some tens of meters deep. Observations show that the skin temperature is invariably a few tenths of a degree cooler than the water a few millimeters below the surface, even during periods of weak winds and strong insolation.

To explain the cool skin, we examine the energy balance of a millimeter thick layer at the ocean surface. The energy balance for this layer differs from that described in Section 9.1 where we examined the surface energy balance in the context of the heat budget of the entire ocean mixed layer. The difference arises because virtually all of the shortwave radiation is absorbed in the ocean mixed layer, while less than 10% is absorbed in the upper millimeter. Since the surface latent and sensible heat fluxes and the net longwave radiation fluxes are typically negative, there is a net heat loss in this millimeter-thick skin layer, even though the ocean mixed layer may be heating due to solar radiation. The net heat loss in the thin surface layer requires a flux of heat from the upper ocean. On both sides of the interface, the atmosphere and ocean are typically in turbulent motion. However upon approaching the interface, turbulence is suppressed and the interface is a strong barrier to the turbulent transport between the ocean and atmosphere. Therefore on both sides of the interface the required heat transfer is accomplished by molecular conduction. To balance the large heat loss at the surface by molecular conduction, the temperature gradient just below the surface must be sufficiently large. Since there is a large heat reservoir in the ocean, the surface skin temperature must drop to accommodate the required heat flux from the ocean interior. This results in a cool skin that is a few tenths of a degree cooler than the ocean temperature a millimeter below the surface.

The diffusive sublayer beneath the ocean surface is characterized by a thickness δ, which is given by (e.g., Krauss and Businger, 1994)

$$\delta = \left(\frac{\kappa\, t^*}{\rho_l\, c_p} \right)^{1/2} \tag{11.2}$$

where κ is the thermal conductivity. The *surface renewal time scale*, t^*, is the residence time of small eddies which are renewed intermittently after random times of contact with the evaporating surface. The surface renewal time scale is a function of the turbulent velocity scale u^*, the surface roughness length z_0, and the net surface heat flux in the surface layer, F_{Q0}^{net}.

The temperature drop across the molecular sublayer, ΔT, which is the bulk-skin temperature difference, is given by (Liu and Businger, 1975)

$$\Delta T \propto F_{Q0}^{net} \left(\frac{t^*}{\rho_l\, c_p\, \kappa} \right)^{1/2} \tag{11.3}$$

Typical nighttime values of ΔT are 0.3°C, although values may exceed 1°C under some extreme conditions. During the daytime there is significant variability in ΔT that depends on the amount of solar insolation, ocean turbidity, and the magnitude of the wind. While such a small value of ΔT may seem insignificant, use of the bulk temperature instead of the skin temperature to calculate the surface sensible and latent heat fluxes from (9.10) and (9.11) can result in errors in the computed fluxes that exceed 10%. Errors of this magnitude may be large enough to change even the sign of the net surface heat flux and could significantly modify boundary layer and convective processes. Because of the Clausius-Clapeyron relationship, small errors in surface temperature result in larger errors in the latent heat flux particularly when the surface temperature is high.

When evaporation exceeds precipitation, a “salt skin” forms on the surface, analogously to the cool skin. This tends to occur in the subtropical oceanic regions (see Section 9.2) where precipitation is very light. The cooling at the surface combined with the higher salinity causes an increase in density at the surface; depending on the stratification of the ocean below the sublayer, this may promote convective stirring of the ocean mixed layer. A dramatic case of the salt skin occurs over open water in the sea ice pack during winter, when surface sensible and latent heat fluxes may exceed several hundred W m^{-2} as the cold air that has been modified over the ice pack streams over open leads or polynyas. Since open water in the ice pack is very near the freezing point, it would seem that any skin cooling would immediately result in surface freezing. However, because of the high evaporation from the surface, surface skin salinity is expected to exceed 80 psu, allowing a cool skin to exist at a temperature below the freezing temperature that corresponds to the mixed layer salinity. This increase in surface salinity will decrease the surface saturation vapor pressure and thus q_{v0}, reducing the surface latent heat and evaporative fluxes.

11.3 Surface Density Changes and the Ocean Mixed Layer

The top few tens of meters of the ocean are usually observed to be well mixed, with uniform temperature and salinity and thus neutral buoyancy (Figure 11.2). This layer is mixed primarily by frictional effects from the wind, although buoyancy effects (e.g. nighttime surface cooling, enhanced turbulent fluxes) can contribute to the mixing. The ocean mixed layer that mediates heat and water exchanges with the atmosphere.

The processes that govern the evolution of the ocean mixed layer are illustrated in the context of a simple bulk mixed layer model, the term "bulk" implying that the temperature and salinity are explicitly specified to be constant over the depth of the mixed layer. Neglecting horizontal advection, the time-dependent equations for temperature and salinity of the ocean [after (3.56) and (3.58)] can be integrated vertically over the mixed layer depth, h_m, to obtain

$$h_m \frac{\partial \theta_m}{\partial t} + u_{ent}\,\Delta\theta = \frac{1}{\rho_m c_p}\left[-\kappa\left(\frac{\partial\theta}{\partial z}\right)_{h_m} + F_{Q0}^{rad} + F_{Q0}^{SH} + F_{Q0}^{LH} + F_{Q0}^{PR} - \left(F_{Q0}^{SW}\right)_{h_m}\right] \quad (11.4)$$

$$h_m \frac{\partial s_m}{\partial t} + u_{ent}\,\Delta s = -D_s\left(\frac{\partial s}{\partial z}\right)_{h_m} + \frac{1}{\rho_m}\left[\left(-\rho_l \dot{P}_r - \rho_S \dot{P}_S + \dot{E} - \dot{R}\right)s_m + \rho_i \frac{d h_i}{dt}\left(s_m - s_i\right)\right] \quad (11.5)$$

where the component terms of the net surface heat and salinity fluxes are described in Sections 7.4 and 7.5. The terms $\Delta\theta = h_{m-} - h_m$ and $\Delta s = s_{m-} - s_m$ represent the jumps in potential temperature and salinity across the base of the mixed layer and u_{ent} is the entrainment velocity at the base of the mixed layer. The subscript h_m on the $\partial\theta/\partial z$ and $\partial s/\partial z$ terms denotes the vertical derivative at the base of the mixed layer, and $(F_{Q0}^{SW})_{h_m}$ denotes the shortwave flux that passes through the base of the mixed layer.

Local changes in the mixed layer depth are determined from[1]

$$\frac{\partial h_m}{\partial t} = u_{ent} \tag{11.6a}$$

The entrainment velocity, u_{ent}, indicates the rate of mixed layer thickening that is caused by the turbulent entrainment of fluid across h_m and by its incorporation into the mixed layer. The entrainment velocity is determined from the conservation of turbulence kinetic energy in the mixed layer, and responds to the surface momentum and buoyancy fluxes. In effect, it is assumed that a fraction of the energy input by the wind is diffused downward and used to mix fluid up from below the mixed layer, the remainder being dissipated. The entrainment velocity can be written (following Chu and Garwood, 1989) as

$$u_{ent} = \frac{c_1 u_*^3 - c_2 (F_{B0}/\rho) h_m}{h_m (\alpha g \Delta T - \beta g \Delta s)} \tag{11.6b}$$

where u_* is the surface friction velocity (9.15), $c_1 \sim 2$ and $c_2 \sim 0.2$. If the surface is cooling then the resulting convective instability can provide mixing energy; if it is being heating some of the mechanical mixing energy is used to overcome this stability effect. Therefore, the depth of the mixed layer will increase with increasing surface wind speed and decreasing surface buoyancy flux.

The ocean mixed-layer characteristics vary in response to varying surface buoyancy and momentum forcing. In the following, we examine the characteristics of the ocean mixed layer at the two thermodynamic extremes of the world ocean: the tropical western Pacific "warm pool" and the ocean mixed layer beneath Arctic sea ice.

11.3.1 Tropical Ocean Warm Pool

The warmest sea surface temperatures in the global ocean occur in the tropical western Pacific Ocean and equatorial Indian Ocean, which is termed the *warm pool* (Figure 11.3). In the western Pacific warm pool, skin temperatures may reach as high as 34°C. The surface winds are typically light (averaging about 5 m s^{-1}), precipitation is very heavy ($\dot{P}_r - \dot{E} \sim 2$ m yr^{-1}), and there is net surface heating (see Figure 7.11). As a result, the mixed layer is relatively shallow, with a mean depth of about 30 m. Figure 11.4 shows a representative vertical structure of the western Pacific warm pool mixed layer. The large amount of precipitation generates a distinct, stable mixed layer near the surface,

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[1] See Kantha and Clayson (1998) for a detailed description of determination of the mixed layer depth.

about 40 m deep (indicated by the depth of the constant density layer). The nearly isothermal layer extends to a depth of about 100 m.

The low surface wind speeds and the presence of a relatively fresh isohaline layer just below the surface implies that the heat content of the ocean mixed layer and the skin temperature are very sensitive to heating from above. Processes occurring in the warm pool are illustrated by the time series shown in Figure 11.5. Between days 318–326, surface wind speed averaged 4 m s^{-1} and a diurnal cycle is seen in skin temperature with an amplitude of 1–2°C. The depth of the mixed layer is very shallow during this period, deepening to about 20 m with the nocturnal cooling but shallowing to about 4 m during daytime. During the subsequent period 327–331, wind speed averaged 7 m s^{-1} and a progressive increase in mixed layer depth is seen. Between days 332–337, wind speed diminishes to about 3 m s^{-1} and steady warming resumes. The period 350-370 is characterized by much stronger winds, with peak values reaching 15 m s^{-1}. As a result, the fresh layer is breached and the ocean mixed layer depth decreases markedly to 75 m as a result of entrainment. During this period the amplitude of the diurnal cycle in skin temperature and mixed layer depth is virtually zero.

11.3.2 Ice/ocean Interactions

The exchange of heat, freshwater and momentum between the atmosphere and ocean is modulated strongly by the presence of a sea ice cover. Ice growth and melting causes buoyancy fluxes that affect the ocean mixed layer below the ice (see Section 9.3). Since the momentum flux from the atmosphere to the ocean is damped by the presence of ice, buoyancy fluxes play a more important role in maintaining the mixed layer than does wind.

Figure 11.6 shows a time series of monthly-averaged salinity and temperature profiles in the ocean beneath the ice in the Arctic Ocean. The salinity of the mixed layer is relatively low, between 30 and 31 psu. This low value arises from the large amount of river runoff ($\dot{R}$) into the Arctic basin. Mixed-layer salinity reaches a maximum in early spring and a minimum in late summer, the annual cycle of salinity dominated by the freeze-melt cycle of the sea ice. The mixed layer salinity exhibits its most rapid changes during the summer melt period, from late June to early August. During the remainder of the year, changes in the mixed layer salinity are more gradual and act to salinize the mixed layer. This salinization is caused both by the salinity flux at the mixed layer base and by the brine rejection and drainage that occurs during ice growth.

Variations in the freezing temperature occur over the annual cycle occur in response to the salinity variations. The mixed-layer temperature does not depart much from the freezing temperature, although small departures occur during winter as a result of constitutional supercooling (see Section 5.7) and during summer owing largely to absorption of solar radiation. Only about 4% of the incident shortwave radiation is transmitted to the ocean, and this is transmitted through the thinnest ice and open water areas.

The mixed-layer depth is modulated by mechanical stirring (arising from the relative velocities of the ice and the ocean) and buoyancy effects. Variations in density occur primarily as a result of variations in salinity and hence the mixed layer depth is determined by the salinity profile. During autumn, winter, and spring, the mixed layer becomes more saline as a result of brine rejection and drainage due to ice growth. This increases the density of the ocean just beneath the ice, which produces static instability and enhances the turbulent activity in the mixed layer, causing it to deepen to a maximum depth of 60 m in mid-May. During the melt season, fresh water from the melting snow and ice enters the mixed layer and the mixed layer warms due to the increased flux of solar radiation. The warming and freshening stabilize the water column and cause the mixed layer to retreat to a minimum depth of approximately 15 m in mid-July.

The Arctic Ocean mixed layer overlies a very stable density structure down to a depth of about 200 meters, arising from a strong increase of salinity with depth. The density structure is a significant factor for sea ice growth because the stable stratification implies that the convective depth to which cooling occurs before freezing commences at the surface is generally not large (see Section 5.6). Therefore, heat and salt from the waters underneath the top 200 meters generally do not upwell to the surface. Hence the surface waters, influenced also by the freshwater river inflows, remain relatively fresh and cold.

11.4 Instability and Mixing in the Ocean Interior

Upper water masses are modified by heat and freshwater exchanges with the atmosphere or from solar radiation transmitted through the atmosphere. By contrast, intermediate and deep water masses are modified only by advective or diffusive exchanges with other water masses; there are no internal sources of heat or salinity. Instability can be generated in the ocean interior by mixing processes, which can lead to rapid water mass modification and generation of deep oceanic convection and associated circulations.

In considering mixing and instability in the ocean interior, it is useful to employ isopycnal (constant density) surfaces, rather than the usual cartesian *x-y* surfaces. An isopycnal surface can also be thought of as a neutral surface (Figure 11.7), such that if a water parcel moves a small distance isentropically along an isopycnal surface (*i.e.,* at constant θ and s), then the parcel will not experience any buoyant restoring forces.

Denoting the two-dimensional gradient operator along an isopycnal surface by ∇_i, we can write (after McDougall, 1984)

$$\rho^{-1}\,\nabla_i \rho = \beta'\,\nabla_i s - \alpha'\nabla_i \theta + \gamma'\nabla_i p \tag{11.7}$$

Values of the thermal expansion coefficient α', saline contraction coefficient β', and compressibility coefficient γ' [2] are defined as (following Gill, 1982)

$$\alpha' = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial \theta}\right)_{sp} = \alpha\left(\frac{\partial \theta}{\partial T}\right)^{-1}_{s,p} \tag{11.8a}$$

$$\beta' = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial s}\right)_{\theta p} = \beta + \alpha'\left(\frac{\partial \theta}{\partial s}\right)_{T,p} \tag{11.8b}$$

$$\gamma' = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial p}\right)_{\theta,s} = \gamma + \alpha'\left(\frac{\partial \theta}{\partial p}\right)_{T,s} \tag{11.8c}$$

where we have from (1.31a), (1.31b) and (1.31d) expressions for the usual coefficients

$$\alpha = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_{s,p} \qquad \beta = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial s}\right)_{T,p} \qquad \gamma = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial p}\right)_{T,s}$$

If a water parcel moves isentropically along an isopycnal surface, implying conservation of θ and s, we can write from (11.7)

$$\rho^{-1}\,\nabla_i \rho = \gamma'\,\nabla_i p \tag{11.9}$$

which implies

$$\alpha'\,\nabla_i \theta = \beta'\,\nabla_i s \tag{11.10}$$

Equation (11.10) can be used to define uniquely the isopycnal surface. By analogous arguments, we can also show that

$$\beta'\,\frac{\partial s}{\partial t} = \alpha'\,\frac{\partial \theta}{\partial t} \tag{11.11}$$

for the time derivatives on an isopycnal surface.

The isopycnal surface is the natural reference frame from which to interpret convective instabilities. Velocity of the fluid perpendicular to

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[2] The speed of sound, c_s, is defined in terms of the adiabatic compressibility coefficient γ' as $c_s^2 = 1/(\rho\gamma')$.

the isopycnal surfaces is associated with instability. As described in Section 7.1, the stability of a parcel of water is determined by the buoyancy frequency, N^2

$$g^{-1} N^2 = \alpha\left(\frac{dT}{dz} + \Gamma_{\mathrm{ad}}'\right) - \beta \frac{ds}{dz}$$

Using (11.8a) and (11.8b), we can write the corresponding definition of buoyancy frequency as

$$g^{-1} N^2 = \alpha' \frac{d\theta}{dz} - \beta' \frac{ds}{dz} \tag{11.12}$$

where the density is regarded as a function of θ, s, and p (rather than T, s, and p). Neutral stability $N^2 = 0$ therefore corresponds to $\alpha' \, d\theta/dz = \beta' \, ds/dz$.

To further interpret the criterion for instability, we can expand the expression (11.12) by using the conservation equations for heat (3.6) and salt (3.11). The conservation equations for heat and salt including only the source terms associated with molecular diffusion are written as

$$\frac{\partial\theta}{\partial t} + u_i \frac{\partial\theta}{\partial x_i} = \frac{\partial}{\partial x_i} K_i \frac{\partial\theta}{\partial x_i} + \frac{\kappa}{\rho c_p} \frac{\partial^2\theta}{\partial x_3^2}$$

$$\frac{\partial s}{\partial t} + u_i \frac{\partial s}{\partial x_i} = \frac{\partial}{\partial x_i} K_i \frac{\partial s}{\partial x_i} + D_s \frac{\partial^2 s}{\partial x_3^2}$$

where we have written the turbulent flux terms as eddy diffusivity terms, following (3.50) and (3.51). We assume that the eddy diffusivity coefficient, K, is identical for heat and salt, although the value of the eddy diffusivity coefficient in the vertical direction, K_3, is different from the values in the horizontal directions, $K_1 = K_2$. The conservation equations can be written in the reference frame of an isopycnal surface by formulating an orthogonal coordinate system consisting of the isopycnal surface and a coordinate that is perpendicular, called the *diapycnal coordinate*. Since the typical slope of an isopycnal surface in the ocean is approximately 10^{-3}, we can consider the isopycnal coordinate system to consist simply of a small rotation of the cartesian coordinate system. Thus the scale factors are unity and the conservation equations have exactly the same form in the isopycnal reference frame as in the cartesian reference frame. We can therefore write the conservations of heat and salt in the isopycnal reference frame as

$$\frac{\partial\theta}{\partial t} + u_i \nabla_i \theta + u_3 \frac{\partial\theta}{\partial x_3} = \nabla_i K_i \nabla_i \theta + \frac{\partial}{\partial x_3} K_3 \frac{\partial\theta}{\partial x_3} + \frac{\kappa}{\rho c_p} \frac{\partial^2\theta}{\partial x_3^2} \tag{11.13}$$

$$\frac{\partial s}{\partial t} + u_i \nabla_i s + u_3 \frac{\partial s}{\partial x_3} + = \nabla_i K_i \nabla_i s + \frac{\partial}{\partial x_3} K_3 \frac{\partial s}{\partial x_3} + D_s \frac{\partial^2 s}{\partial x_3^2} \tag{11.14}$$

where the notation ∇_i represents the horizontal gradient along the isopycnal surface, u_i represents the horizontal velocity components along the isopycnal surface, u_3 is the vertical velocity in the diapycnal direction, and $\partial/\partial x_3$ represents the derivative in the diapycnal direction.

Multiplying (11.14) by β' and subtracting α' times (11.13) gives

$$\left(u_3 - \frac{\partial K_3}{\partial x_3}\right) \frac{N^2}{g} = K_i \left(\alpha' \nabla_i^2 \theta - \beta' \nabla_i^2 s\right) + K_3 \left(\alpha' \frac{\partial^2 \theta}{\partial x_3^2} - \beta' \frac{\partial^2 s}{\partial x_3^2}\right) + \left[\frac{\alpha' \kappa}{\rho c_p} \frac{\partial^2 \theta}{\partial x_3^2} - \beta' D_s \frac{\partial^2 s}{\partial x_3^2}\right] \tag{11.15}$$

where

$$g^{-1} N^2 = \alpha' \frac{d\theta}{du_3} - \beta' \frac{ds}{du_3} \tag{11.16}$$

and we have ignored horizontal variations in the eddy diffusivity coefficient. Note that use of the isopycnal coordinate system results in elimination of the time derivative and advection terms, providing a principal motivation for using this coordinate system. The term (u_3 - $\partial K_3/\partial x_3$) is the diapycnal velocity, and gives the speed at which the isopycnal surface moves through the fluid. Note that spatial variations of diffusivities are referred to as *pseudovelocities* since they appear in conservation equations as advective terms but do not appear in the continuity equation. The diapycnal velocity is a direct result of the various mixing processes represented on the right-hand side of (11.15). The first term on the right-hand side describes the instabilities that arise from horizontal mixing on an isopycnal surface. The second term represents an instability associated with diapycnal turbulent mixing. The last term in (11.15) is associated with double-diffusive convection.

More specifically, the first term of the right-hand side of (11.15) represents the isopycnal mixing of parcels of water with different values of θ and s. A typical value of K_i in the ocean is 1000 m^2 s^{-1}. The instabilities associated with this term can be elucidated as follows. Taking the isopycnal divergence of the identity $\alpha' \nabla_i \theta = \beta' \nabla_i s$ (11.10), we obtain (following McDougall, 1984)

$$\begin{aligned} K_i \left(\alpha' \nabla_i^2 \theta - \beta' \nabla_i^2 s\right) &= -K_i \left|\nabla_i \theta\right|^2 \left[\frac{\partial \alpha'}{\partial \theta} + 2 \frac{\alpha'}{\beta'} \frac{\partial \alpha'}{\partial s} - \frac{\alpha'^2}{\beta'^2} \frac{\partial \beta'}{\partial s}\right] \\ &\quad - K_i \nabla_i \theta \cdot \nabla_i p \left[\frac{\partial \alpha'}{\partial p} - \frac{\alpha'}{\beta'} \frac{\partial \beta'}{\partial p}\right] \end{aligned} \tag{11.17}$$

The first term on the right-hand side of (11.17) is called the *cabbeling instability*. When two parcels of water are mixed along an isopycnal surface, the density of the mixture increases because of the nonlinearity of the equation of state of seawater. The cabbeling parameter is given by

$$\frac{\partial \alpha'}{\partial \theta} + 2\,\frac{\alpha'}{\beta'}\,\frac{\partial \alpha'}{\partial s} - \frac{\alpha'^2}{\beta'^2}\,\frac{\partial \beta'}{\partial s}$$

A typical value of the cabbeling parameter is 10^{-5} K^{-2}. Figure 11.8 shows isopycnals near 0°C and 34.6 psu. Two different water types, A and B, are identified with different values of temperature and salinity, but having the same density (σ = 27.8). If the two water types mix, their final temperature and salinity will be a mass-weighted average of the two different water types (analogous to Section 6.4). It is easily shown that the properties of the mixture lie on the straight line AB, and thus the mixture is more dense than either of the original water types. Consequently, any such mixture will tend to sink. Thus the cabbeling instability arises from the nonlinear relation between density, temperature, and salinity, particularly at low temperatures.

The second term on the right hand side of (11.17) represents the *thermobaric instability*, which arises from the variation of the thermal expansion and saline contraction coefficients with pressure. The thermobaric parameter is given by

$$\frac{\partial \alpha'}{\partial p} - \frac{\alpha'}{\beta'}\,\frac{\partial \beta'}{\partial p}$$

A typical value of the thermobaric parameter is 2.6x10^{-12} K^{-1} Pa^{-1}. Table 11.1 shows the variation of α and β with pressure at T = 2°C and s = 35 psu. The compressibility of cold water is generally greater than that of warm water, so a cold parcel displaced downward could in principle become heavier than its surroundings. The variation of the saline contraction coefficient with pressure is small, and hence its contribution to generating thermobaric instability is much less. The thermobaric instability is illustrated in Figure 11.9. Along a neutral surface, p and θ both vary so that $\nabla_i\theta \cdot \nabla_i p \neq 0$. Consider two parcels of fluid with different temperatures on an isopycnal surface. If $\nabla_i\theta \cdot \nabla_i p > 0$, the parcels move below the neutral surface, and if $\nabla_i\theta \cdot \nabla_i p < 0$ the parcels rise above it. If the parcels are moved back to their original positions, no irreversible effects will have occurred. However the mixing of the two parcels together off the isopycnal surface (cabbeling) is irreversible and hence consolidates the vertical movement of the parcels away from the neutral surface by thermobaricity.

Table 11.1 Variation of α and β with pressure, for a temperature of 2°C and salinity of 35 psu. Note that β differs from β' by less than 0.3% and that $\alpha'=\alpha$ (T/θ) $[c_p(p_r,\theta)/c_p(p,T)]$ where p_r is a reference pressure of 1 bar.

p (bar)	α (10^{-7} K^{-1})	β (10^{-6} psu^{-1})
0	781	779
100	1031	767
200	1269	758
300	1494	747
400	1707	737
500	1907	728
600	2094	719

Thermobaricity and cabbeling are processes caused by lateral mixing along isopycnal surfaces that result in vertical advection because the equation of state for seawater is nonlinear. Thermobaricity arises from the variation of α'/β' with pressure on an isopycnal surface, while cabbeling arises from the dependence of α'/β' on θ and s along an isopycnal surface. While cabbeling and thermobaricity are effective at producing diapycnal downwelling (or upwelling) of water, they are ineffective at producing vertical diffusion of tracers. Over most of the global ocean, thermobaricity and cabbeling are weak. However, where the slope of the isopycnal surfaces is significant, the downwelling due to cabelling is quite large. Cabbeling and thermobaricity are largest in the North Atlantic and oceans surrounding Antarctica, causing contributions to the diapycnal downwelling velocity of the order of -1×10^{-7} m s^{-1}. Thermobaricity is generally smaller than cabbeling except in the Antarctic Circumpolar Current where it is equivalent in magnitude and also of the same sign.

The last term in (11.15) is associated with *double-diffusive instability*. When a one-component fluid is stably stratified, no vertical velocities are generated by buoyancy forces. In a multi-component fluid such as seawater, in which density is determined by concentration of the solute in addition to the temperature of the solution, buoyant instabilities can arise simply from the different values of the diffusivity of heat and the diffusivity of the scalar concentration. The ensuing instability in the ocean is referred to by convention as double-diffusive instability. The thermal diffusivity of seawater, $\kappa/\rho c_p \sim 1.4 \times 10^{-7}$ m^2 s^{-1} is two orders of magnitude larger than the diffusivity of salt in water, $D_s \sim 1.1 \times 10^{-9}$ m^2 s^{-1}. Therefore, heat is transferred by diffusion more rapidly than salt. Even if the mean density distribution is hydrostatically stable, the difference in molecular diffusivities allows instability to be initiated. For double diffusion instability to occur in the ocean, the temperature and salinity gradients across the isopycnal surface must have the same sign, since temperature and salinity act in opposite directions to influence density. There are two kinds of double diffusive convection: diffusive staircases, in which relatively fresh, cold water overlies warmer, saltier water; and salt finger staircases, in which warm, salty water overlies cooler, fresher water.

Consider a layer of warm, salty water overlying a cooler, fresher layer of water, both with equal densities, so that $T_1 > T_2$, $s_1 > s_2$, and $\rho_1 = \rho_2$. The warm salty layer will lose heat to the cooler, fresher layer faster than it will lose salt. By this double diffusion process, the density of the upper layer will increase and the density in the lower layer will decrease. Buoyancy forces cause the upper, denser layer to sink. This results in *salt fingering*, with the instability arising from the lower diffusive flux of salinity. In this case, long narrow convection cells or *salt fingers* rapidly form (Figure 11.10). Individual salt fingers are a few millimeters across and up to 25 cm long. After the salt fingers form, lateral diffusion occurs between the fingers and produces a uniform layer. The process may start again at the two new interfaces so that eventually a number of individually homogeneous layers develop with sharply defined interfaces in terms of temperature and salinity. These layers may be meters to tens of meters thick, separated by thinner interface zones of sharp gradients of temperature and salinity, giving the vertical temperature and salinity profiles a step-like appearance. Double diffusion occurring by this mechanism was first observed below the outflow of relatively warm, salty Mediterranean Water into the Atlantic.

Now consider the situation of cold, less salty water overlying warmer, saltier water, with a stable stratification. The instability is driven by the larger vertical flux of heat relative to salt through the "diffusive" interfaces, which are in turn kept sharpened by the convection in the layers. This gives rise staircases with regularity of the steps and systematic increase of both s and θ with depth. Such layers have been observed in the Arctic Ocean, Weddell Sea, and the Red Sea (Figure 11.11).

It is now recognized that salt fingering and related processes can make a significant contribution to vertical mixing within the oceans. It seems that double-diffusive staircases could in principle be observed at any interface between water masses and in the vicinity of all thermoclines. The buoyancy flux ratio for salt fingers, R_ρ, is given by

$$R_\rho = \frac{\alpha' \Delta\theta}{\beta' \Delta S} \tag{11.18}$$

It appears that salt fingers do not occur if $R_\rho > 0.6$. Salt fingers are not always observed even if conditions are favorable if more vigorous processes are operative, such as turbulent mixing.

In regions where there are significant horizontal density gradients (sloping isopycnal surfaces), thermobaricity, cabbeling, and double diffusion can produce significant diapycnal velocities in the ocean. These processes influence water mass transformation in the ocean interior, particularly at the boundaries of water masses. However, in regions of the ocean where horizontal density gradients are small, processes such as thermobaricity, cabbeling, and double diffusion are often negligible relative to other processes.

11.5 Oceanic Convection and Deep Water Formation

Thermodynamic mechanisms driving oceanic deep convection are caused by the convective instabilities described in the preceding section. Any mechanism that causes a denser water mass to lie over a lighter water mass will give rise to such an instability. In addition to the thermodynamic mechanisms, there are dynamical instabilities that contribute to deep convection. Dynamical instabilities in the atmosphere and ocean are the dynamic response on a rotating earth to thermodynamic imbalances. These dynamical instabilities include Kelvin-Helmholtz instability, baroclinic instability, centrifugal instability and symmetric instability[3]. Dynamical instabilities exchange water mass in the vertical direction, which reduces the stratification, making the ocean more susceptible to thermodynamic instabilities.

Deep convection in the open ocean is observed to occur in the Labrador, Greenland, Weddell, and Mediterranean Seas. These regions are the principal source regions for deep and intermediate waters of the world's oceans. No deep water is formed in the North Pacific under the present climate regime, primarily because the static stability is too large and surface waters too fresh even at the freezing point to sink below a few hundred meters.

Deep convection in the open ocean occurs through the following sequence of events.

i) A background cyclonic gyre circulation (counterclockwise) (Figure 11.12a) develops in the ocean in response to the surface winds. The cyclonic gyre is associated with rising motion in the ocean (upward vertical velocity). The gyre forms an elongated "dome" of isotherms, isohalines and isopycnals in the center of the cyclonic gyre, on a horizontal scale of about 100 km. This reduces the vertical stability of water columns within the gyre.

[3] A discussion of dynamical instabilities in the ocean is beyond the scope of this book; for further information, the reader is referred to *Deep Convection and Deep Water Formation in the Oceans* (1991) by Chu and Gascard.

ii) Formation of a preconditioning pool (Figure 11.12b) occurs by a combination of density increase in the upper ocean due to surface cooling and salinization plus strong surface winds, leading to mixed layer penetration into the intermediate-water dome. This creates a background of low static stability within the gyre. Formation of an oceanic frontal system by hydrodynamical instability also supports preconditioning.

iii) Mixed-layer penetration into the intermediate water dome may result in a mixed water mass with an increased density via cabbeling. A more efficient mode of mixing occurs via mixing of two different water masses across a front (associated with baroclinic instability) This can result in a dramatic increase in the depth of the mixing.

iv) Narrow plumes with width on the order of 1 km develop, with strong downward motion (Figure 11.12c) reaching as high as 10 cm s^{-1}. The plumes broaden as they descend through lateral entrainment. The mode of convection seems to be nonpenetrative rather than penetrative. In other words, mixing occurs in such a way as to keep the density structure a continuous function of depth.

v) The plumes rapidly mix properties over the preconditioned pool, forming a vertical "chimney" of homogeneous fluid. When the surface forcing ceases, convective overturning declines sharply and the predominantly vertical heat transfer gives way to horizontal transfer.

vi) The chimneys eventually breakup due to mushrooming around bottom topographical features, vertical shear of the mean circulation, dynamical instabilities, and mixing associated with internal waves. The horizontal spreading of the deep water is an efficient way for anomalous wintertime atmospheric features to extend their effect far below the surface layer of the ocean and far from their location of origin.

The best documented open-ocean convection occurs during most winters in the Gulf of Lions of the western Mediterranean Sea. During February, strong wind outbursts of cold continental air blow from the north-northwest over the cyclonic gyre in the western Mediterranean, cooling the surface by evaporation. In this region, the surface buoyancy flux results in a gradual erosion of the thermocline by a deepening surface mixed layer. In the center of the preconditioned dome, intense convection can occur although it does not usually reach the ocean bottom.

North Atlantic Deep Water is formed in the Greenland Sea. The circulation of the Greenland Sea is characterized by cyclonic upper-layer flow (Figure 11.13). Relatively warm and saline water is carried northward by the Norwegian-Atlantic Current. One branch of this current curls cyclonically around the Greenland Sea, thus transporting salt into the region. During early winter, the formation of ice and the associated brine rejection increase mixed layer salinity at freezing point temperatures and reduce the stability of the underlying weakly stratified waters. In the later part of winter, strong surface winds generate ice export and cause deepening of the exposed mixed-layer. Thus a pool of dense water is generated in the central Greenland Sea that is several 100 m deep and 100 km wide. Within this pool, convection begins to develop in late winter in response to favorable atmospheric heat and freshwater fluxes.

Deep ocean convection also occurs sporadically in the Southern Ocean. Strong cooling at the surface of polynyas drives the convection throughout the winter period, leading to substantial deep water formation. The marginal stability of the Southern Ocean water column leaves it susceptible to ocean overturning, but the strong ocean heat flux acts to limit sea ice growth and minimizes the salinization associated with ice formation. Surface cooling, cyclonic circulation in the ocean, decrease in pycnocline strength, or decreasing the oceanic heat flux all tend to destabilize the stratification during winter, thus increasing the possibility of deep convection.

In addition to sinking in the open ocean, deep convection can also occur near an ocean boundary, such as a continental shelf (Figure 11.14). Near-boundary convection in the high latitudes occurs in the following sequence of events:

i) Dense water forms over the continental shelf, due to surface cooling and salt rejection that occurs during ice formation. The width of the shelf seems to be positively correlated with the amount of deep water formed. Providing that the depth of the reservoir is sufficiently shallow (500 m in the Antarctic, 50–100 m in the Arctic), the resulting vertical convection yields cold, salty water at depth on the shelf.
ii) A horizontal density gradient parallel to the coast produces local circulations that are favorable for the reservoir to empty. The horizontal density gradient may be induced by variations in shelf geometry.
iii) Mixing with an off-shore water mass increases the density. While mixing does not appear to be a necessary element of the near-boundary convection, it appears consistently in the observations.
iv) Dense, salty water on the shelf descends the slope under a balance of Coriolis (a “force” arising from the Earth’s rotation), gravity, and frictional forces. The thermobaric effect may also contribute to the sinking.

Near-boundary convection has been observed along coastal regions surrounding Antarctica, the coastal regions of the Arctic Ocean, and in the coastal regions surrounding the Greenland and Norwegian Seas.

Outside the polar regions, boundary deep convection has been observed in the Red Sea and the Northern Adriatic Sea.

The lower 2000 m of the global ocean is dominated by Antarctic Bottom Water (AABW), with temperature below 2°C. AABW forms both by open ocean convection and along coasts. Polynyas (Section 10.1) play a major role in the formation of AABW. Along the coasts, large amounts of sea ice form within the coastal polynyas which is continually swept away by coastal winds, resulting in further ice formation and salt rejection. The AABW formation arises from the mixing of shelf and slope water across the shelf break. The dense water formed at this front sinks along the continental slope to the deep and bottom depths to a thin plume of concentrated AABW. During the mid-1970's, a large polynya in the Weddell Sea formed each winter, associated with vigorous oceanic convection that eliminated the sea ice cover. It has been suggested (Gordon 1982) that there may be a tradeoff between open ocean and shelf production of deep water in the Southern Ocean. Advection of sea ice northward acts to destabilize the shelf region (where more ice forms) while simultaneously stabilizing the open ocean region (where the ice melts). Hence, conditions favoring open ocean convection coincide with unfavorable conditions on the shelves and vice versa.

The North Atlantic Deep Water, formed by deep convection in the Greenland Sea, is warmer and saltier (2.5°C, 35 psu) than Antarctic Deep Water that forms in the Weddell Sea (-1.0°C, 34.6 psu). More than half the ocean's volume is filled with cold water from these two sources. The rates of production and flow of deep water has been inferred from the distributions of tritium (^{3}H) in the decades after atmospheric nuclear weapons tests deposited large amounts of tritium in the northern North Atlantic and Arctic Oceans. It has been hypothesized that the formation of North Atlantic deep water is especially vulnerable to changes in the high latitude surface salinity flux (*e.g.,* runoff, precipitation, evaporation, sea ice transport).

11.6 Global Thermohaline Circulations

The *thermohaline circulation* is basically an overturning of the oceans in the meridional-vertical plane. In contrast to the quasi-horizontal wind-driven gyres, which are constrained to limited ranges of latitude and depth, the thermohaline-driven overturning cells are global in scale. The thermohaline circulation is that part of the large-scale ocean flow driven by diapycnal fluxes, rather than directly by the wind. The term thermohaline circulation is used increasingly to apply not only to the direct response to surface buoyancy fluxes but also to flows whose characteristics are altered significantly by upwelling or mixing. The interhemispheric and interbasin water mass exchanges associated with the thermohaline circulation have an important influence on the distribution of properties in the deep ocean and on global-scale heat and fresh water transports. Although thermohaline processes in the ocean are commonly considered separately from wind-driven processes, many aspects of the ocean thermohaline and wind-driven circulations are inextricably linked by nonlinearities in the system and cannot be easily separated.

Surface heating at the equator and cooling at high latitudes implies a thermohaline circulation associated with the meridional (latitudinal) density differences. However the circulation associated with this overturning remains weak since the heating near the equator occurs only in the shallow mixed layer. For an efficient thermohaline "engine", expansion must take place at higher pressure, and compression at lower pressure (Figure 11.15a). Inclusion of salinity effects shows overall that freshening in high latitudes counteracts the cooling effect. Figure 11.15b shows a schematic of the upper ocean thermohaline circulation. An efficient thermohaline circulation seems to be restricted to the upper layers in tropical and subtropical regions. This upper thermohaline circulation is of minor importance compared to the wind-driven circulation.

In the deep ocean (below the thermocline), where the ocean is not influenced directly by winds, thermohaline circulations are very important, if not very efficient. Local thermohaline circulations occur in certain marginal seas, associated with warm waters rendered dense by their high salinity resulting from evaporation. Water from the Mediterranean, made dense by evaporation in the eastern basin, flows westward below the surface along that sea out through the Strait of Gibraltar into the Atlantic. There it spreads westward across the ocean, being identifiable as a high salinity tongue between about 500 and 3000 m. In the Red Sea, evaporation increases the salinity in the summer and then winter cooling of the saline upper waters causes it to sink to mid-depth and flow out and southward in the Indian Ocean as a high salinity tongue between 500 and 2500 m.

The Atlantic Ocean thermohaline circulation is illustrated in Figure 11.16. The abbreviations for the water masses shown in Figure 11.16 are given in Table 11.2. In the high latitudes of the North Atlantic, surface water is advected northward and cools as it enters the polar seas. Upper North Atlantic Deep Water (UNADW) is formed from a mixture of Labrador Sea Water (LSW) and Mediterranean Overflow Water (MOW), along with entrained Antarctic Intermediate Water (AAIW). AAIW forms as the relatively cool, fresh water of the Southern Ocean flows northward and sinks, spreading at a level near 1000 m below the main thermocline. Deep water is formed on the shelves of the Arctic Ocean. The Eurasian Basin Deep Water (EBDW) is advected into the

North Atlantic through the Fram Strait. Lower North Atlantic Deep Water (LNADW) is formed from Nordic Seas Overflow Water (NSOW), Denmark Strait Overflow Water (DSOW), and EBDW that mixes with intermediate water plus modified AABW. The LNADW comprises the bottom water north of 45°N, while AABW comprises the bottom water in the rest of the global ocean. The deep water that moves southward across the equator thus originated as AABW, EBDW, NSOW, DSOW, LSW, AAIW, and MOW. The Circumpolar Deep Water (CDW) formed initially from NADW, but becomes modified by circulation in the Antarctic Circumpolar Current System and transits in the Pacific and Indian Oceans, undergoing mixing along the way. The circulation is completed by weak but widespread upwelling in the low- and mid-latitudes, and also by the wind-driven overturning that drives the Antarctic circumpolar current.

Table 11.2 Abbreviations of water masses used in Figure 11.16 and elsewhere in the text (after Schmitz, 1996).

Abbreviation	Full name
AABW	Antarctic Bottom Water
AAIW	Antarctic Intermediate Water
CDW	Circumpolar Deep Water
DSOW	Denmark Strait Overflow Water
EBDW	Eurasian Basin Deep Water
LNADW	Lower North Atlantic Deep Water
LSW	Labrador Sea Water
MOW	Mediterranean Outflow Water
MNADW	Middle North Atlantic Deep Water
NACW	North Atlantic Central Water
NADW	North Atlantic Deep Water
NSOW	Nordic Seas Overflow Water
UNADW	Upper North Atlantic Deep Water

A schematic of the global ocean thermohaline circulation, or conveyor belt, is shown in Figure 11.17. The Antarctic Circumpolar Current System is seen to be the conduit for transporting NADW to the Indian and Pacific Oceans, either as pristine NADW or modified CDW. The source of NADW has multiple paths, with upwelling and water mass conversion occurring in numerous locations. Radioactive tracer measurements show that the ocean thermohaline circulation brings deep ocean water into contact with the atmosphere every 600 years or so. The overturning is driven by both buoyancy forces generated in high-latitude oceans and the mechanical process driven by the wind stress in the region of the Antarctic Circumpolar Current System. While the Atlantic branch of the global thermohaline circulation is fairly well known, substantial uncertainties remain particularly regarding the thermohaline flow patterns near the equator and the deep circulation in the North Pacific.

Since the density differences driving the global ocean thermohaline circulation are small, slight changes in surface salinity at high latitudes caused by precipitation, evaporation, ice melt, or river runoff can have a large effect on the circulation strength. Paleoceanographers hypothesize that the shutdown of the North Atlantic thermohaline circulation was a key feature of past Northern Hemisphere glaciation. Melt water discharge at the end of the glaciation provided enough fresh water to insulate the deep ocean from the atmosphere on a large scale, turning off the North Atlantic thermohaline circulation and returning Europe to ice age conditions. A modern instance of freshwater capping in the North Atlantic has been documented, called the *great salinity anomaly*. Salinity records suggest that a large discharge of Arctic sea ice led to patch of fresh upper ocean water, which circulated around the subpolar gyre during the 1960's and 1970's, which caused deep convection locally to cease. The insulating effect of such low salinity patches causes reduced heat and moisture flux to the atmosphere and cooler conditions in Europe. Buoyancy-driven overturning circulations are vulnerable to shutdown by relatively small increases in high-latitude freshwater fluxes. An overturning that includes a significant degree of mechanical forcing by Southern Hemisphere winds over the Drake Passage may be less vulnerable to such changes.

Notes

Surface renewal theory is described in *Atmosphere-Ocean Interaction* (1994, Chapter 5) by Kraus and Businger and in *Small-scale Processes in Geophysical Flows* (1998, Chapter 4) by Kantha and Clayson.

Characteristics of the ocean mixed layer and the associated physical processes are described in *Small-scale Processes in Geophysical Flows* (1998, Chapter 2) by Kantha and Clayson. An overview of the Arctic Ocean mixed layer and ice/ocean interactions is given by Holland et al. (1997).

Mixing in the ocean is described by Turner (1981). The roles of thermobaricity and cabbeling in water mass conversion are described by McDougall (1987). Kantha and Clayson (1998, Chapter 7) give a thorough description of double-diffusive convection.

An extensive overview of oceanic deep convection is given in *Deep Convection and Deep Water Formation in the Oceans* (1991) by Chu and Gascard. Additional reviews are given by Killworth (1983) and Schott et al. (1994).

A discussion of the relative roles of thermohaline and wind-driven processes in the general circulation of the ocean is given in *Atmosphere-Ocean Dynamics* (1982) by Gill.

Warren (1981) provides an overview of the deep ocean thermohaline circulations and Schmitz (1995) gives a recent review of the interbasin-scale thermohaline circulation.

Problems

1. Using the data given in problem **9.2** for noon, determine the error in the surface upwelling longwave flux and the sensible and latent heat fluxes if the bulk sea surface temperature (ocean mixed layer temperature) of 29°C was used instead of the skin temperature.

2. Assume that the incident solar flux at the surface is 500 W m^{-2}. Use the following spectral extinction coefficients for the ocean:

spectral interval (μm)	kv, ext (m^{-1})	spectral weight (%)
0.25-0.69	0.126	46
0.69-1.19	17	32.6
1.19-2.38	1000	18.1
2.38-4.00	10000	3.3

a) Calculate the radiative heating rate in the ocean at depths 1, 5, 10, 20, 50, and 100 m that is associated with solar radiation (using Beer's Law).

b) What is the buoyancy frequency, N^2, after 6 hours of solar heating, assuming that no further mixing takes place? Assume that the net heating of the ocean mixed layer is determined solely by the solar radiation flux, and that the temperature of the mixed layer is 30°C and its depth is 100 m.

3. In the marginal ice zone of the North Atlantic, compare the contributions of mechanical mixing energy and the buoyant mixing to the ocean mixed layer entrainment heat flux, F_{Q0}^{ent}. What percent error in the ocean mixed layer entrainment heat flux would be made by incuding only the contribution from mechanical mixing energy? Assume the following values: $u* = 0.017$ m s^{-1}; $\rho = 1028$ kg m^{-3}, $h_m = 75$ m; $dh_i / dt = 0.1$ m day^{-1}; $(\dot{P} - \dot{E}/\rho) = -0.005$ m day^{-1}; $F_{Q0} = -500$ W m^{-2}; $s_m = 34.6$ psu; $T_m = -1.89$ °C; $T_{m-} = 0$ °C; $s_m = 34.6$ psu; $s_{m-} = 34.95$ psu; $\alpha = 0.025 \times 10^{-3}$ K^{-1}, $\beta = 0.79 \times 10^{-3}$ psu^{-1}; c_p=4186 J kg^{-1} K^{-1}.

A value $c_1 = 2$ is appropriate for a high-latitude deep mixed layer.