12.1 Planetary Radiation Balance

The *luminosity* of the Sun, $L_o \sim 3.9 \times 10^{26}$ W, is the total rate at which energy is released by the Sun. From the value of luminosity, it can be estimated that the Sun emits radiation at an equivalent black body temperature of about 6000 K. Since space is essentially a vacuum and energy is conserved, the amount of radiation which reaches a planet is inversely proportional to the square of the distance between the planet and the Sun (*inverse square law*). The distance between a planet and the Sun is determined from the mean planet-Sun distance and the eccentricity and obliquity of the orbital plane. The axial tilt, which is the angle between the axis of rotation and the normal to the plane of orbit, influences the seasonal and latitudinal variation of insolation. The *solar constant*, S, is defined as the amount of solar radiation received per unit time and per unit area, perpendicular to the Sun's rays at the top of the atmosphere, at mean Earth-Sun distance. The solar constant can be evaluated from the inverse square law to be

$$S = \frac{L_o}{4\pi \bar{d}^2}$$

where $\bar{d}$ is the mean Earth-Sun distance. The solar constant has been monitored by satellite and is found to be about 1370 ± 4 W m^{-2}.[1]

The solar flux at the top of the atmosphere, F_{TOA}^{SW}, is then given by

$$F_{TOA}^{SW} = S \left(\frac{\bar{d}}{d}\right)^2 \cos Z \tag{12.2}$$

where Z is the solar zenith angle, d is the Earth-Sun distance, and $\bar{d} = 150 \times 10^{11}$ m is its average value.

Expansion of a Fourier series for the squared ratio of the mean Earth-Sun distance to the actual distance is given by

$$\left(\frac{\bar{d}}{d}\right)^2 = \sum_{n=0}^{2} a_n \cos(n\varphi_d) + b_n \sin(n\varphi_d) \tag{12.3}$$

where

$$\varphi_d = \frac{2\pi d_n}{365}$$

and d_n is the day number (January 1 ≡ 0; December 31 ≡ 364). The Fourier coefficients in (7.36) are given by (Spencer, 1971)

n	a_n	b_n
0	1.000110	0
1	0.034221	0.001280
2	0.000719	0.000077

The *solar zenith angle*, Z, is defined as the angle between the vertical direction and the direction of the incoming solar beam (see Section 3.3) and is given by

$$\cos Z = \sin \phi \sin \delta + \cos \phi \cos \delta \cos \psi \tag{12.4}$$

where ϕ is the latitude (ϕ is negative in the southern hemisphere), δ is the solar declination angle, and ψ is the hour angle. The *hour angle* is zero at solar noon and increases by 15° for every hour before or after solar noon. The *solar declination angle* is a function only of the day of the year and is independent of location. It varies from 23°45' on June 21 to -23°45' on December 21, and is zero on the equinoxes. The declination angle can be approximated by

$$\delta = \sum_{n=0}^{3} a_n \cos\left(n\varphi_d\right) + b_n \sin\left(n\varphi_d\right)$$

where the coefficients are given by (Spencer, 1971)

n	a_n	b_n
0	0.006918	0
1	-0.399912	0.070257
2	-0.006758	0.000907
3	-0.002697	0.001480

The daily average insolation calculated using (12.1) is shown in Figure 12.3. The annual average radiation received at the poles is approximately half that received at the equator. A very small annual cycle is seen at the equator, with a slight semi-annual oscillation having maxima at the equinoxes and minima at the solstices. At the pole, direct sunlight is absent for exactly half the year. However, near the summer solstice the daily amount of radiation received at the pole exceeds that received at the equator because the Sun is above the horizon for 24 hours per day at the pole. A slight asymmetry in insolation between the hemispheres is seen, arising from the fact that the Earth is closer to the Sun during the northern hemisphere winter because of the eccentricity of its orbit.

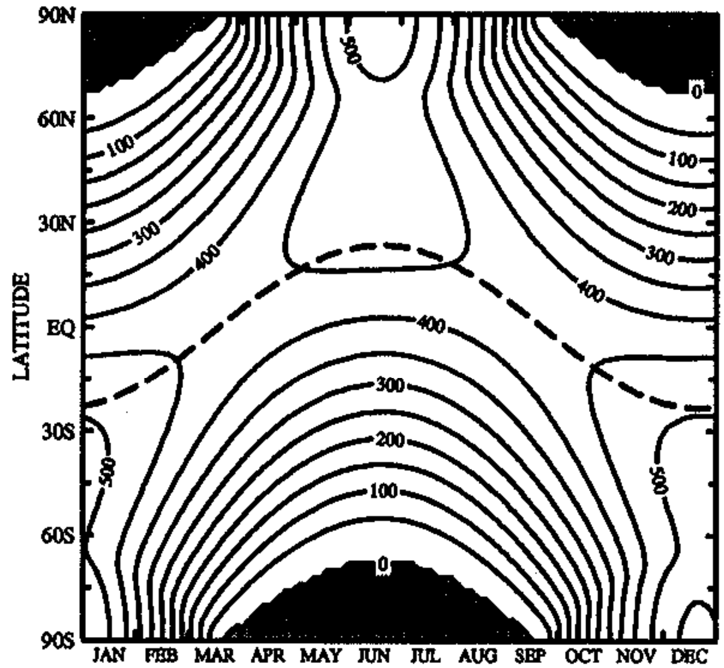

Figure 12.3 Daily average insolation at the top of the atmosphere as a function of latitude and season. Units are W m^{-2}. Dashed line indicates the latitude at which the sun is directly overhead. (From Hartmann, 1994.)